MIMO Channel Estimation with Score-Based Generative Priors learned from Noisy Data

ArXiv Link

Asad Aali[1], Marius Arvinte[1,2], Sidharth Kumar[1], Jonathan I. Tamir[1]

[1]*Chandra Family Department of Electrical and Computer Engineering*, The University of Texas at Austin, Austin, TX

[2]*Intel Corporation,* Hillsboro, OR

Introduction

- Generative models trained on clean data distribution have shown to outperform end-to-end supervised deep learning.
- A large collection of clean training data is prohibitively expensive to acquire.
- Our method approximately learns a generative model of the clean distribution from noisy data.
- We present SURE-Score: a novel loss function that leverages Stein's unbiased risk estimate (SURE) to jointly denoise the data and learn a score function

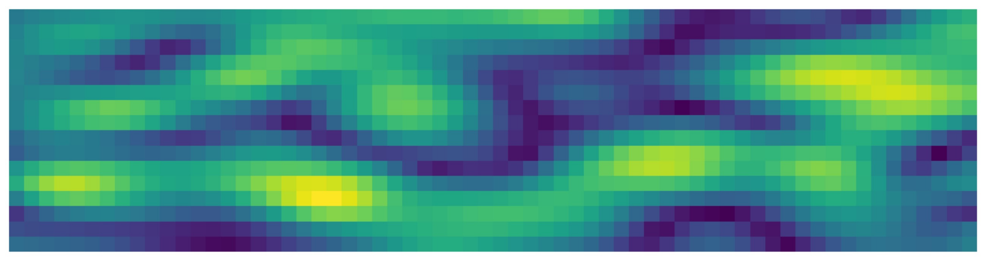
Wireless System Theory

- MIMO forward model: $\mathbf{Y} = \mathbf{HP} + \mathbf{N}$.

$\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$	Channel state information matrix
$\mathbf{p}_i \in \mathbb{C}^{N_t}$	Pilot symbol, P = $(p_0, p_1, \ldots, p_b)$, where $b = \alpha_{pilot} * N_t$
$\sigma^2_{\text{pilot}}\mathbf{I}$	Complex Additive White Gaussian Noise

- Narrowband, point-to-point MIMO communication scenario
- Channel estimation requires estimating H, using the received pilot matrix Y, while having knowledge of the transmitted pilot matrix P

$$\widetilde{H} = H + w, w \sim N\left(0, \sigma_w^2 I\right)$$



Example Clustered Delay Line (CDL-C) channel (magnitude)

Training SURE-Score

$$\mathcal{L}(\theta) = \alpha \underbrace{\left(\mathbb{E}_{\widetilde{H}, n_i}\left[\sigma_i^2 \left\| s_\theta\big(\overbrace{\widetilde{H} + \sigma_w^2 s_\theta(\widetilde{H})}^{f_\theta(\widetilde{H})} + n_i\big) + \frac{n_i}{\sigma_i^2}\right\|_2^2\right]\right)}_{\text{Score Loss}} + \underbrace{\left(\mathbb{E}_{\widetilde{H}, w}\left[\big\|\overbrace{\sigma_w^2 s_\theta(\widetilde{H})}^{f_\theta(\widetilde{H}) - \widetilde{H}}\big\|_2^2 + 2\sigma_w^2 \text{div}_{\widetilde{H}}\big(\overbrace{\widetilde{H} + \sigma_w^2 s_\theta(\widetilde{H})}^{f_\theta(\widetilde{H})}\big)\right]\right)}_{\text{Modified SURE Loss}}$$

Where $\text{div}_{\widetilde{H}}\left(\widetilde{H} + \sigma_w^2 s_\theta(\widetilde{H})\right) = tr\left(J_{\widetilde{H} + \sigma_w^2 s_\theta(\widetilde{H})}\right)$

Where α is appropriate scaling applied to score model

$$\underbrace{\mathbb{E}_{H,w}\left[\left\|f_\theta(\widetilde{H}) - \widetilde{H}\right\|_2^2 + 2\sigma_w^2 \text{div}_{\widetilde{H}}\left(f_\theta(\widetilde{H})\right)\right]}_{\text{Regular SURE Loss}}$$

SURE-based denoising with s_θ — Denoising score matching with s_θ

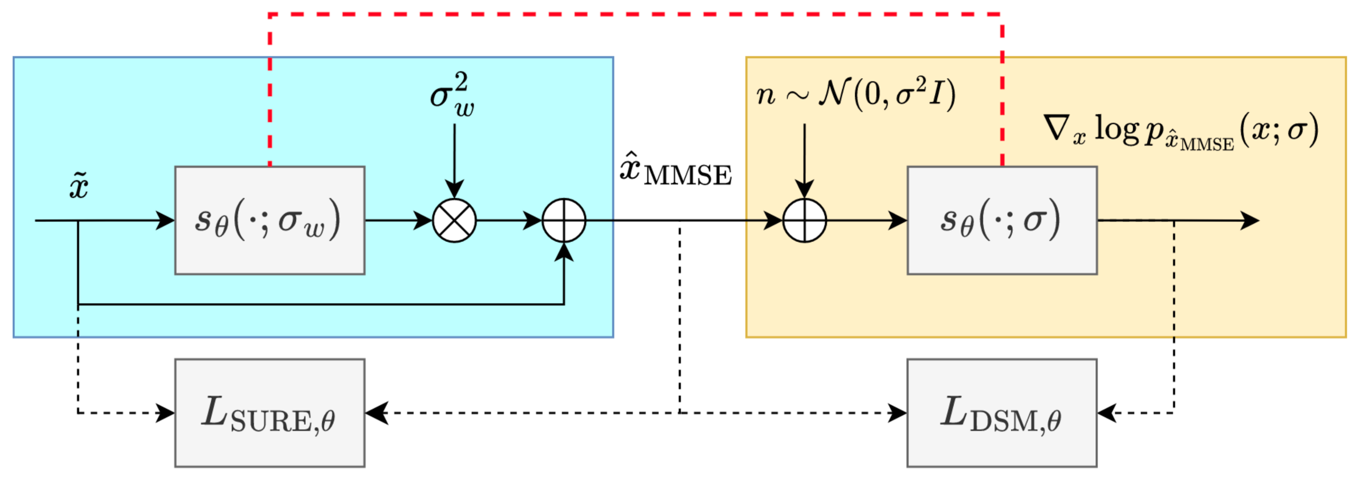


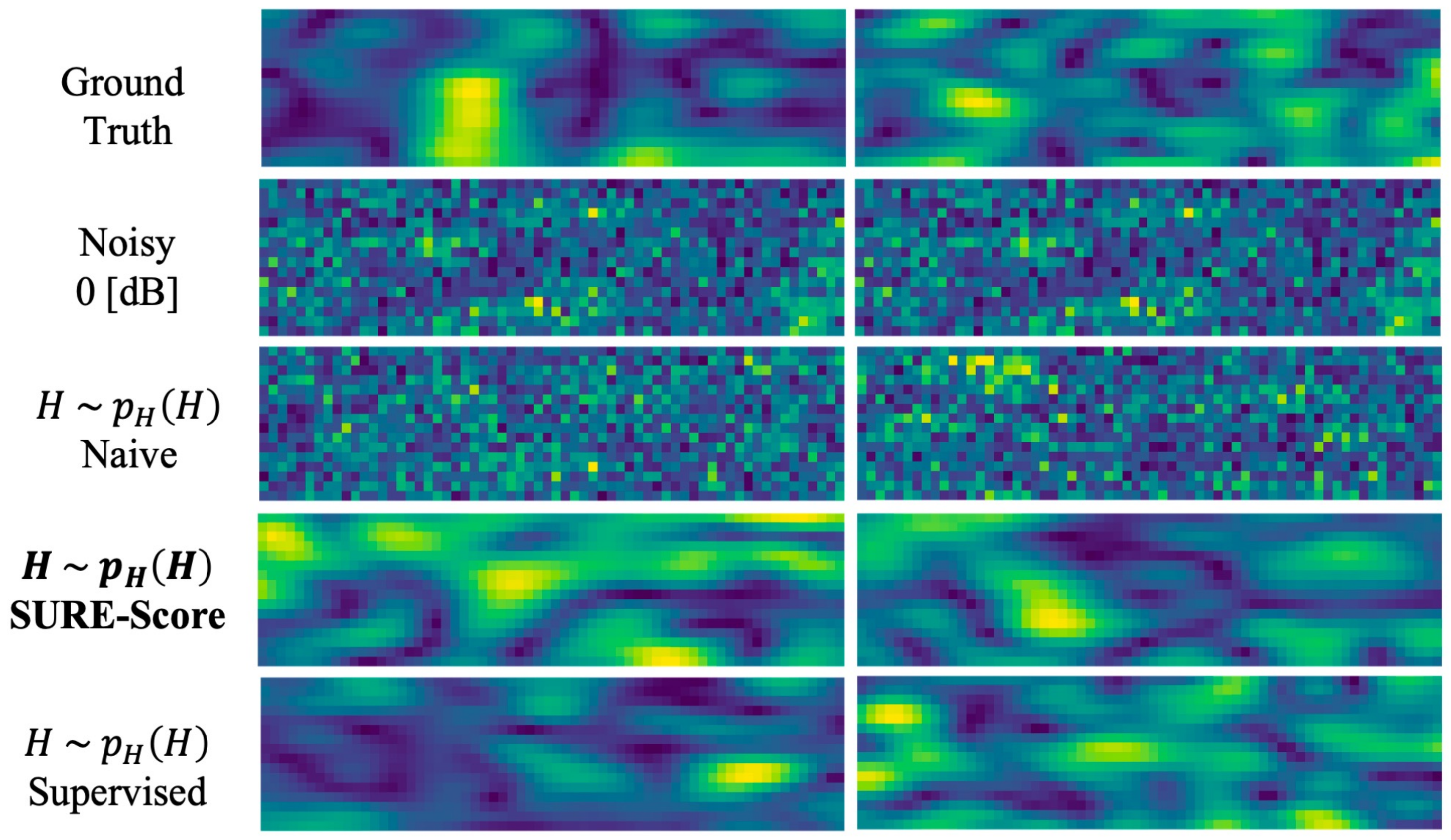
Fig. 1. Flow of SURE-Score during training. The same deep neural network s_θ is used first for denoising and subsequently for denoising score matching.

Generic PyTorch Training Pipeline

Sampling - Annealed Langevin Dynamics

$$\mathbf{H}_{t+1} \leftarrow \mathbf{H}_t + \alpha_t \cdot \nabla \log p_H(\mathbf{H}_t) + \beta_t \cdot \zeta_t.$$

- $\alpha_t \cdot \psi_H(\mathbf{H}_t)$ increases the likelihood of the current sample.
- $\beta_t \cdot \zeta_t$ represents a perturbation to the above process.



Let p_H denote the distribution of MIMO (CDL-C) channels for a stochastic environment.

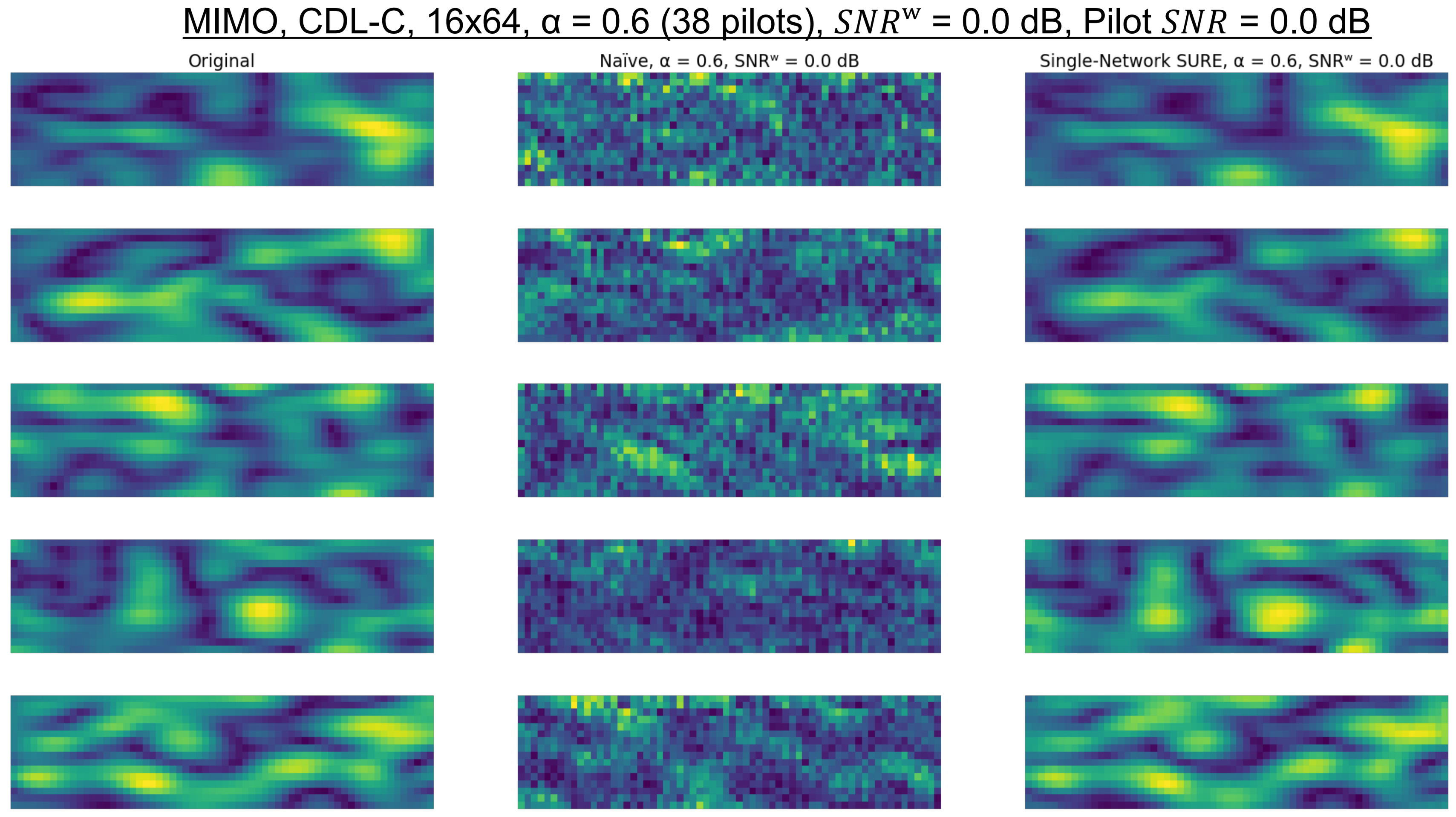
$$\psi_H(\mathbf{H}) = \nabla \log p_H(\mathbf{H}),$$

Fig. 2. Prior sampling for three methods: Naive, SURE-Score at SNR^w 0 dB, and Supervised. Each column is different realization of a CDL-C channel.

Posterior Reconstruction

$$\mathbf{H}_{\text{est},i+1} = \mathbf{H}_{\text{est},i} + \alpha_i \cdot (\nabla \log p_{Y|H}(\mathbf{Y}|\mathbf{H}_{\text{est},i}) + \nabla \log p_H(\mathbf{H}_{\text{est},i})) + \sqrt{2\beta \cdot \alpha_i} \cdot \sigma_{z_i} \cdot \zeta,$$

MIMO, CDL-C, 16x64, α = 0.6 (38 pilots), SNR^w = 0.0 dB, Pilot SNR = 0.0 dB



$$H \sim p(H|Y)$$

Fig. 3.
Naive: Sampling using score model trained directly on noisy channels

Single-Network SURE: Sampling using score model trained on channels denoised using a single network

Posterior Reconstruction – Benchmarking

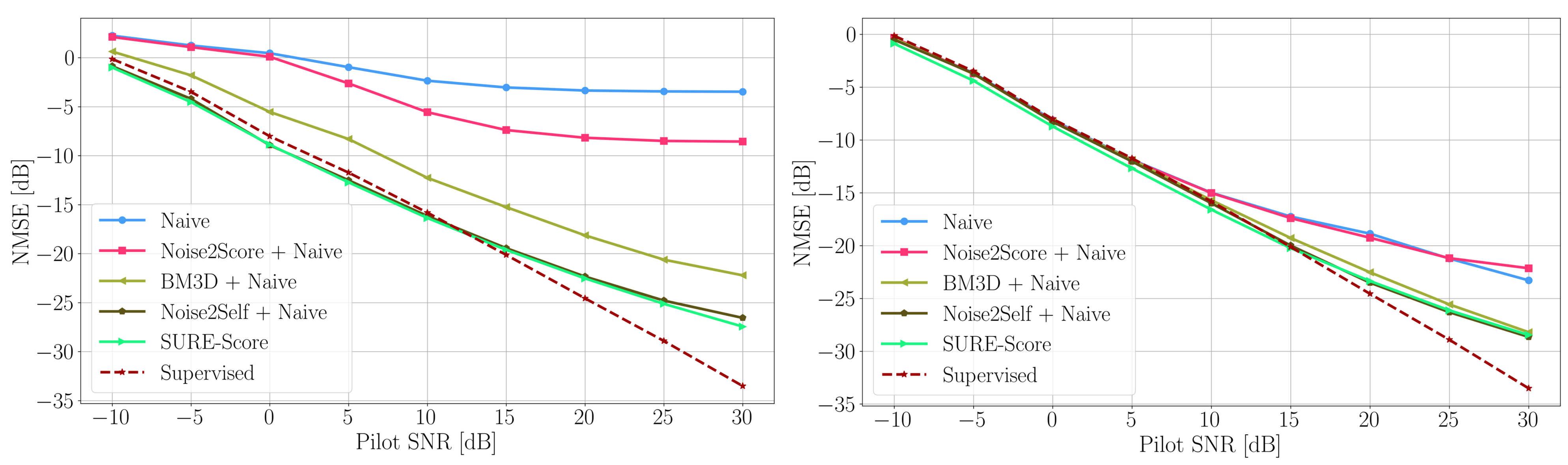


Fig. 3. Channel estimation performance at α = 0.6 (38 pilots) using score models trained on CDL-C channels at SNR^w: 0 dB (left) and 10 dB (right).

Key Takeaways:

- **SURE-Score** performs close to **optimal** with respect to supervised DSM except at higher pilot SNR
- Naive training plateaus in estimation performance because of ***overfitting***
- Noise2Score and BM3D suffer at lower SNR^w and improve at higher SNR
- Performance gap at high pilot SNR likely due to performance limits of MMSE denoiser and finite training data

Generalized SURE-Score

- Goal: Learn the score directly from noisy measurements y

$$y = Ax + n$$

- Where n is a zero-mean Gaussian random vector
- A is full-rank

Methodology:

- Utilize extended SURE principle to obtain unbiased MSE estimate for exponential family noise

$$s(h) = \|x\|^2 + \|h(u)\|^2 + 2\left(Tr\left(\frac{\partial h(u)}{\partial u}\right) + h^T(u)\frac{\partial \ln q(u)}{\partial u}\right)$$

- Use a single-network to jointly denoise the data and learn score-function

Discussion and Conclusion

- Self-supervised techniques can **match** supervised techniques in denoising and inverse problem performance
- Reconstruction performance with and without access to ground truth measurements is equivalent at low SNRs and comparable at high SNRs

- **Next Steps:** Our work currently assumes white Gaussian noise corruption but could be extended to arbitrary exponential families

Selected References

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