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GENERATIVE PRIORS FOR SOLVING INVERSE PROBLEMS FROM NOISY DATA

IFML WORKSHOP 2023

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TEXAS
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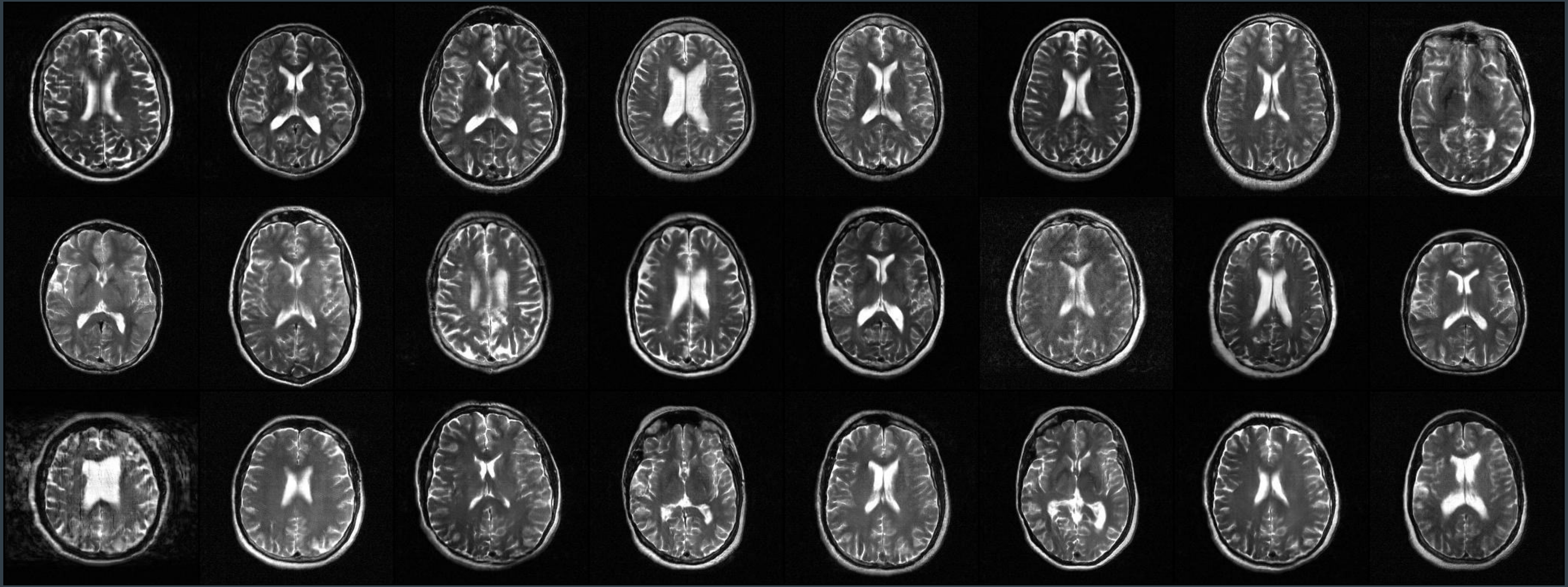
Generative models are powerful image generators

https://thiscatdoesnotexist.com

Generative models are powerful image generators

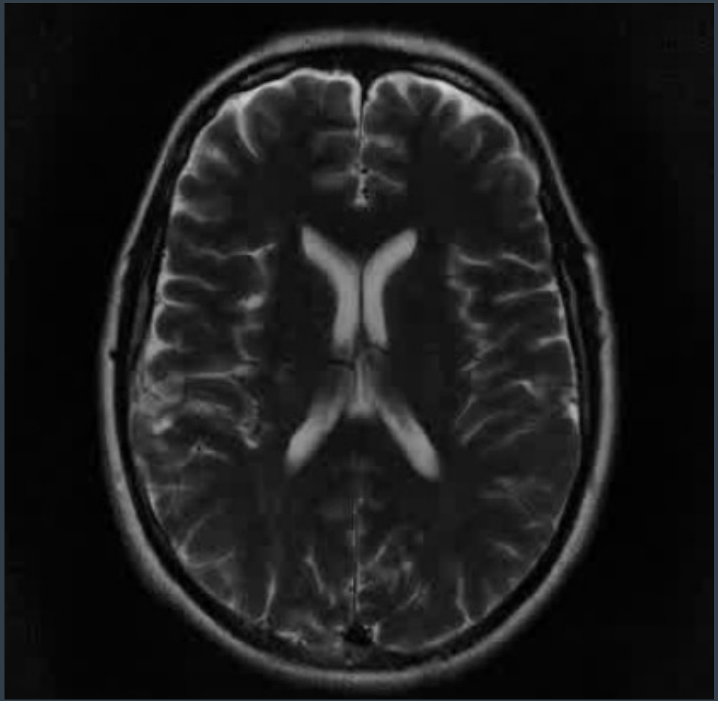


Generative model trained on FastMRI data

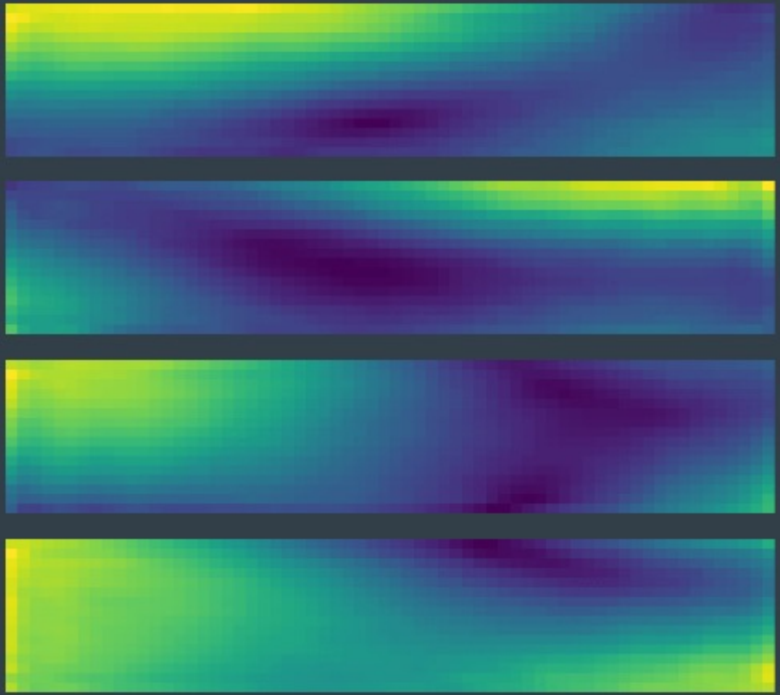
Score models for inverse problems -> $x \sim p(x|y)$

$$y = Ax + \text{noise}$$

Brain FastMRI



Wireless CDL Channels

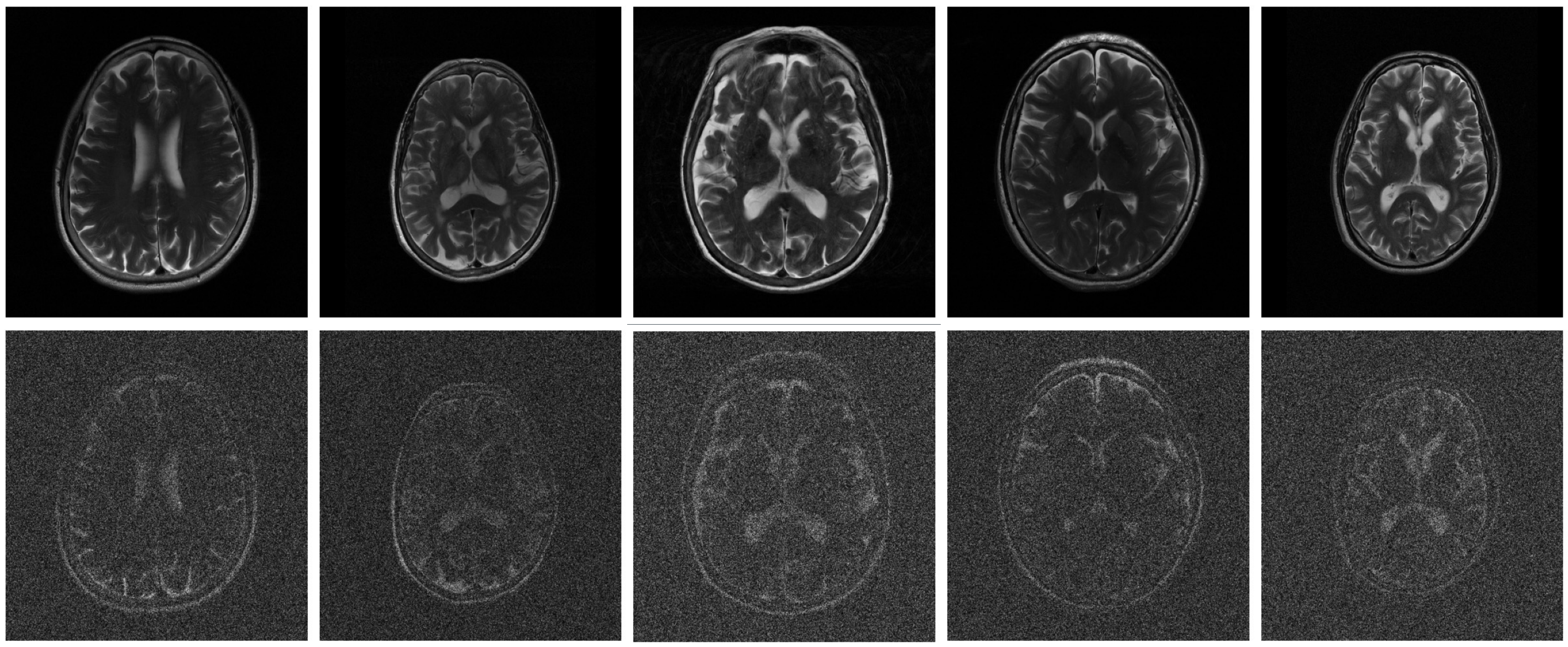


Learning from Noisy Data

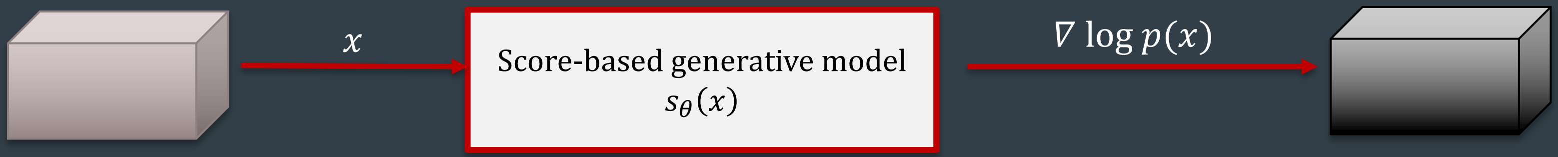
$$\tilde{x} = x + w$$
$$w \sim N(0, \sigma_w^2 I)$$

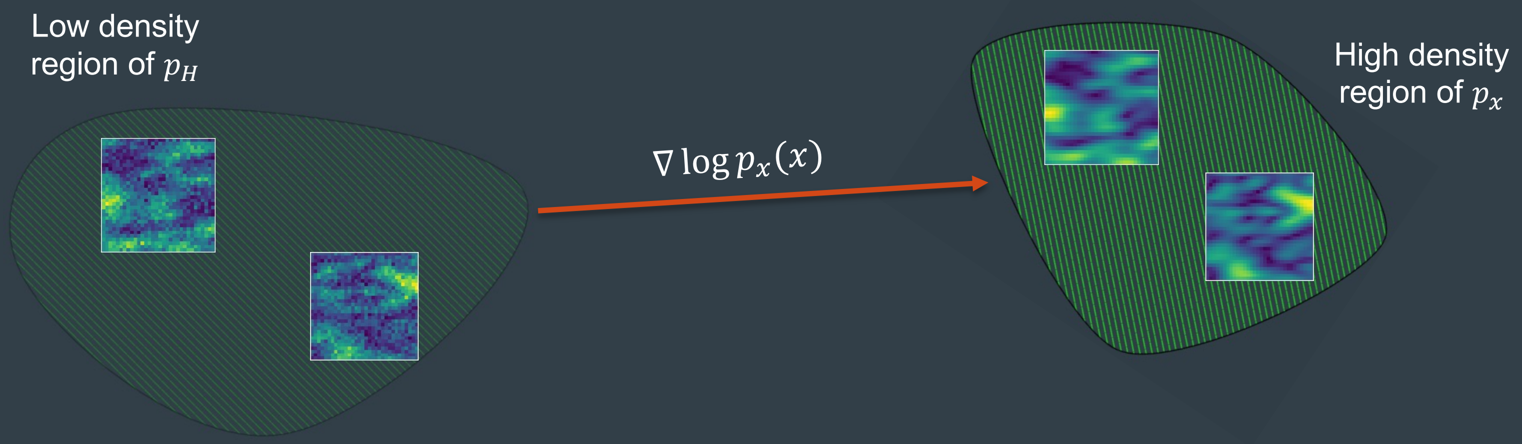
x

$\tilde{x}$
(0 dB)



Score-based generative models [1]





[1] Song, Y., and Ermon, S. "Generative Modeling by Estimating Gradients of the Data Distribution." NeurIPS 2019.

Single-Network SURE-Score – Our Proposed Method

We combine *denoising score matching* and *Stein's Unbiased Risk Estimate (SURE)*

$$\mathcal{L}(\theta) = \alpha \text{ (Denoising Score Matching)} + \text{SURE Denoising}$$

[1] Metzler, Christopher A., et al. "Unsupervised learning with Stein's unbiased risk estimator.", NeurIPS 2018.

Single-Network SURE-Score – Our Proposed Method

We combine *denoising score matching* and *Stein's Unbiased Risk Estimate (SURE)*

$$\mathcal{L}(\theta) = \underbrace{\alpha\left(\mathbb{E}_{\tilde{x},n_i}\left[\sigma_{n_i}^2\left\|\overbrace{s_\theta(\tilde{x}+\sigma_w^2 s_\theta(\tilde{x})+n_i)}^{d_\theta(\tilde{x})}+\frac{n_i}{\sigma_{n_i}^2}\right\|_2^2\right]\right)}_{\text{Denoising Score Matching}} + \underbrace{\left(\underbrace{\mathbb{E}_{\tilde{x},w}[\|\overbrace{\sigma_w^2 s_\theta(\tilde{x})}^{\tilde{x}-d_\theta(\tilde{x})}\|_2^2}_{\text{measurement bias}} + \underbrace{2\sigma_w^2 \operatorname{div}_{\tilde{x}}(\overbrace{\tilde{x}+\sigma_w^2 s_\theta(\tilde{x})}^{d_\theta(\tilde{x})})]}_{\text{divergence}}\right)}_{\text{SURE Denoising}}$$

- Where $\operatorname{div}_{\tilde{x}}(\tilde{x}+\sigma_w^2 s_\theta(\tilde{x})) = tr\left(J_{\tilde{x}+\sigma_w^2 s_\theta(\tilde{x})}\right)$
- Where α is appropriate scaling applied to score loss

[1] Metzler, Christopher A., et al. "Unsupervised learning with Stein's unbiased risk estimator.", NeurIPS 2018.

Experiment 1 – Wireless MIMO CDL Prior Sampling

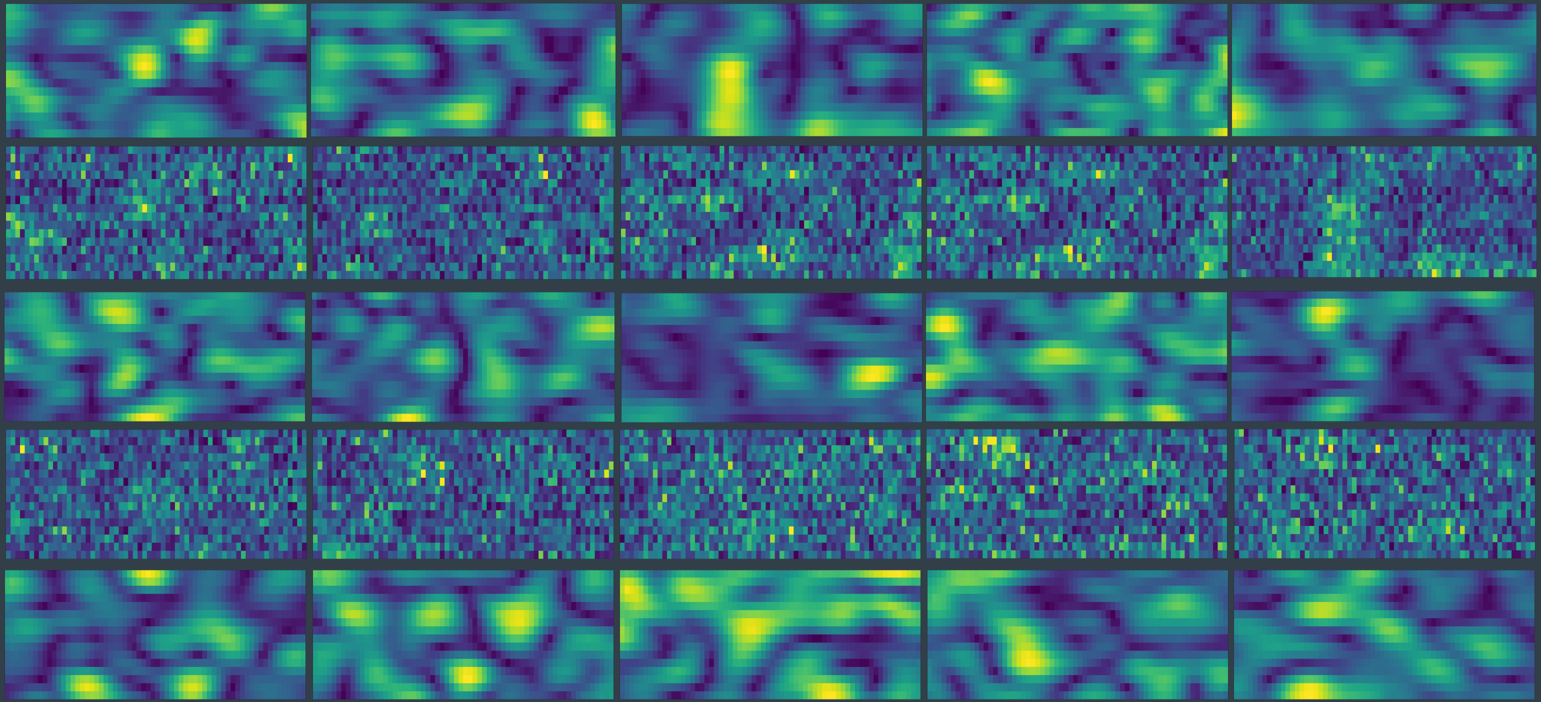
H
True Channels

$\tilde{H}$ (0 dB)

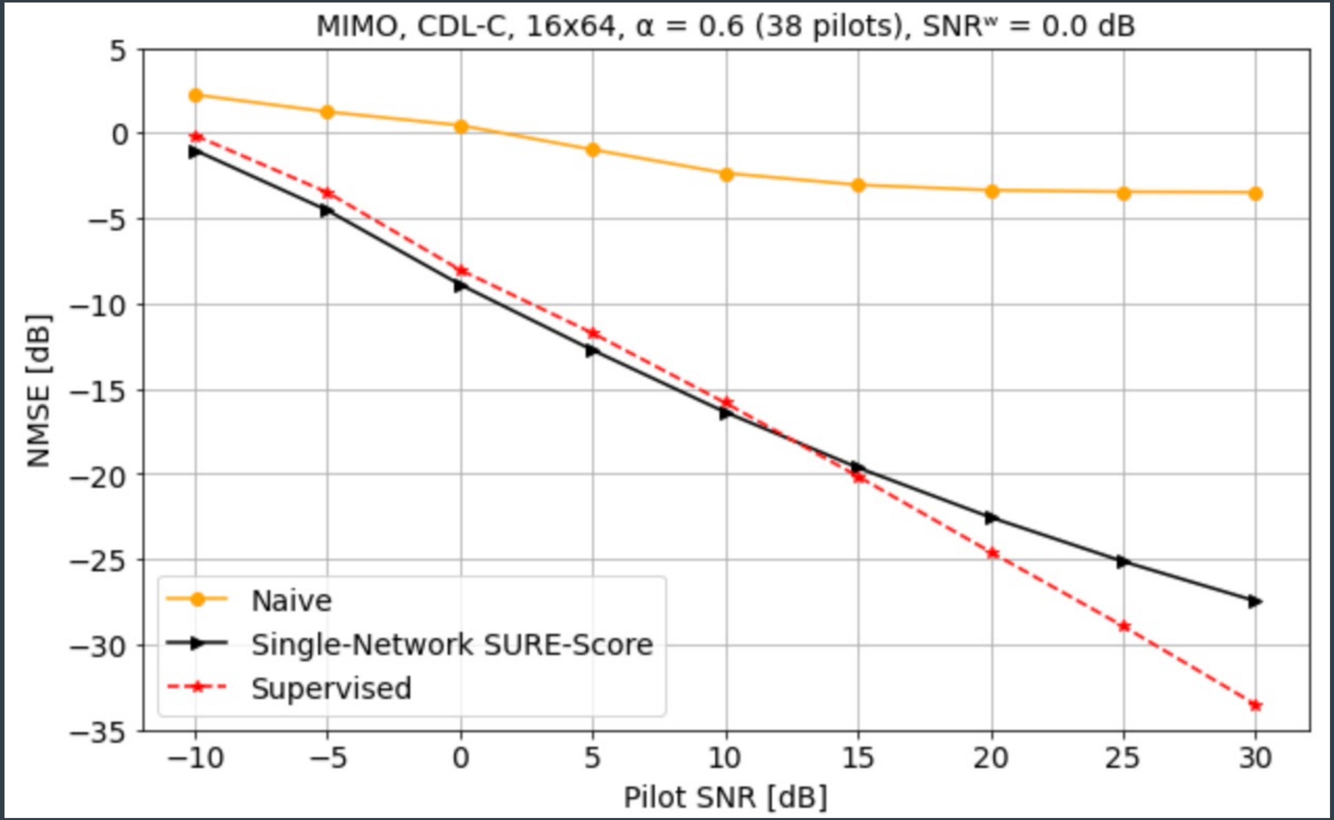
$H \sim p(H)$
Supervised

$H \sim p(H)$
Naive

$H \sim p(H)$
SURE−Score

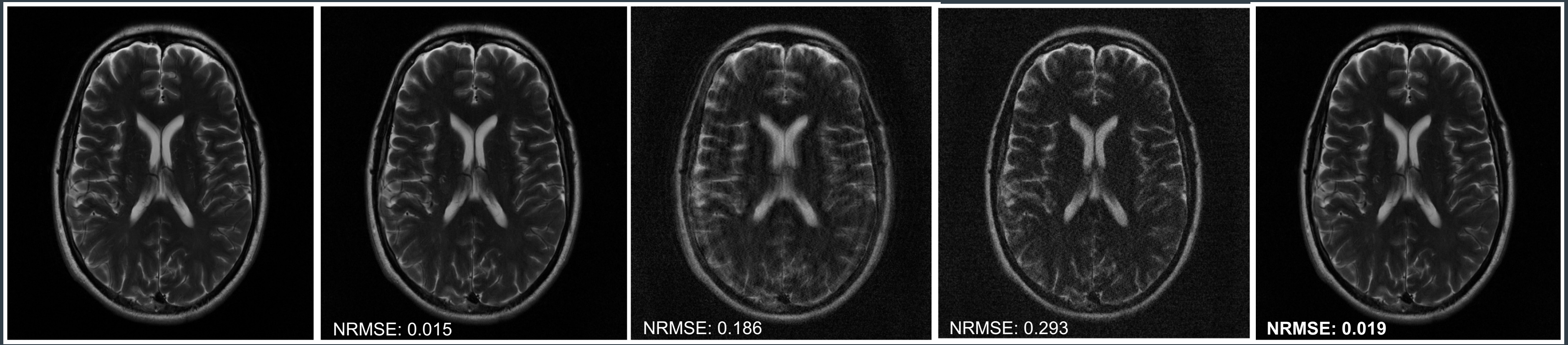


Experiment 2 – Wireless MIMO Posterior Reconstruction



Experiment 3 – FastMRI Posterior Reconstruction

Ground Truth | *Supervised Score Model* | *L2 Regularization* | *Naive* | ***Single-Network SURE-Score***



* Acceleration Factor -> 5

General Score Model Training/Sampling Pipeline

- `score-diffusion-training`
  - Train diffusion models for arbitrary multi-dim data

- `score-diffusion-sampling`

  Prior, posterior sampling for arbitrary forward models

https://github.com/utcsilab

UT Computational Sensing and Imaging Lab

- Joint design of imaging system and software algorithms
- Focus on inverse problems and deep learning applications in MRI
- Work with clinicians to translate work to hospital

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